

# ANALYTIC SOLUTION OF A TEST PROBLEM FOR MODE

Mission Research Corporation 735 State Street Santa Barbara, California 93101

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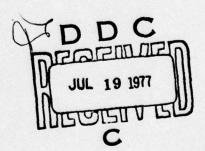
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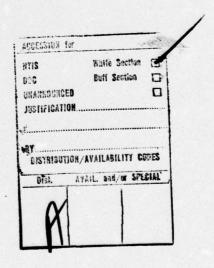
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## SECTION 1

## INTRODUCTION

In the validation of finite difference techniques it is often useful to have a test problem with an analytic solution so that a detailed comparison can be made. In this note we will derive the analytic expression for the fields outside a conducting sphere which is driven by a radial current in space between the surface of the sphere and an infinitesimal distance away from the surface. This current has a cosine distribution in the polar angle and a double exponential time history. We will also present graphs of the analytic results and a comparison with the results of MODE, a 3-D finite difference electromagnetics code with an outer boundary scheme based on a multipole expansion of outgoing waves.

(In MODE, the current was driven in the first radial cell outside the sphere.)

## SECTION 2

## **ANALYSIS**

In solving for the fields outside a conducting sphere of radius a, excited by a radial current density,

$$J(\mathbf{r}) = \hat{\mathbf{r}} I(t) \cos\theta \delta(\mathbf{r}-\mathbf{b}) , \qquad (1)$$

we will use a Green's function technique and contour integration to determine the field time histories. We will first Fourier transform the driver

$$I(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{+i\omega t} I(t) , \qquad (2)$$

where

so

$$I(\omega) = \frac{1}{2\pi} \int_{0}^{\infty} e^{(i\omega - \alpha)t} dt$$
$$-\frac{1}{2\pi} \int_{0}^{\infty} e^{(i\omega - \beta)t} dt$$
(4)

$$=\frac{1}{2\pi}\left\{-\frac{1}{i\omega-\alpha}+\frac{1}{i\omega-\beta}\right\},\qquad (5)$$

(I(t) is in amps/m.)

From the symmetry of the excitation we must only consider  ${
m TM}_{10}$  fields. This portion of the free space transverse Green's function can be written

$$\overrightarrow{G}_{TM10} = i k \overrightarrow{N}_{10}^{1}(\mathbf{r}_{<}) \overrightarrow{N}_{10}^{3}(\mathbf{r}_{>}) , \qquad (6)$$

where

$$\vec{N}_{10}^{i} = \hat{r} \sqrt{2} \frac{z_{1}^{i}(kr)}{kr} Y_{10}(\theta, \phi) + \frac{1}{kr} \left[ kr z_{1}^{i}(kr) \right] \vec{K}_{10}^{i}(\theta, \phi) , \qquad (7)$$

with

$$z_{1}^{1}(kr) = j_{1}(kr)$$

$$z_{1}^{3}(kr) = h_{1}^{(1)}(kr)$$

$$Y_{10}(\theta,\phi) = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$\vec{K}_{10}^{\prime}(\theta,\phi) = \frac{1}{\sqrt{2}}\hat{\theta}\frac{\partial Y_{10}}{\partial \theta} = -\hat{\theta}\sqrt{\frac{3}{8\pi}}\sin\theta$$
(8)

The vanishing of the tangential electric field on the boundary can be ensured by adding to the free space Green's function a term

$$\overrightarrow{G}_{TM10}^{\dagger} = -i k \frac{\left[kaj_{1}(ka)\right]'}{\left[kah_{1}^{(1)}(ka)\right]'} \overrightarrow{N}_{10}^{3}(r_{>}) \overrightarrow{N}_{10}^{3}(r_{<}) .$$
(9)

This part has

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{G'}) - k^2 \vec{G} = 0 , \qquad (10)$$

outside the sphere and a theta component which is the negative of that for the free space Green's function at the surface of the sphere r=a. Since

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} - k^2 \vec{E} = i\omega \mu \vec{J} , \qquad (11)$$

we have

$$\vec{E}(\mathbf{r}) = i\omega\mu \int dV' \overrightarrow{G}(\mathbf{r}, \mathbf{r}') \cdot \vec{J}(\mathbf{r}')$$
 (12)

Inserting Equation 5 into this, we will first look at fields beyond the source, so the electric field will be

$$\vec{E} = \left[ \hat{\mathbf{r}} \sqrt{2} \frac{h_1^{(1)}(kr)}{kr} \sqrt{\frac{3}{4\pi}} \cos\theta - \hat{\theta} \frac{\left[ kr h_1^{(1)}(kr) \right]'}{kr} \sqrt{\frac{3}{8\pi}} \sin\theta \right] \\
\times (-k\omega\mu) \int \mathbf{r'}^2 d\mathbf{r'} d(\cos\theta') d\phi \sqrt{2} \frac{1}{kr'} \sqrt{\frac{3}{4\pi}} \cos\theta' \\
\times \left\{ j_1(kr') - \frac{\left[ ka j_1(ka) \right]'}{\left[ ka h_1^{(1)}(ka) \right]'} h_1^{(1)}(kr') \right\} I_{\omega} \cos\theta' \delta(\mathbf{r'} - \mathbf{b}) . \tag{13}$$

On performing the integral

$$E = \left[\hat{\mathbf{r}} \sqrt{2} \frac{h_1^{(1)}(k\mathbf{r})}{k\mathbf{r}} \sqrt{\frac{3}{4\pi}} \cos\theta - \hat{\theta} \frac{\left[k\mathbf{r}h_1^{(1)}(k\mathbf{r})\right]'}{k\mathbf{r}} \sqrt{\frac{3}{8\pi}} \sin\theta\right] (-\omega\mu) (2\pi)$$

$$\times \left(\frac{2}{3}\right) \sqrt{2} \sqrt{\frac{3}{4\pi}} I_{\omega} b \left\{ j_{1}(kb) - \frac{\left[kaj_{1}(ka)\right]'}{\left[kah_{1}^{(1)}(ka)\right]'} h_{1}^{(1)}(kb) \right\} , \qquad (14)$$

$$E_{\mathbf{r}} = -2 \frac{c\mu b}{r} I_{\omega} h_1^{(1)}(kr) \left\{ \cos \theta , \qquad (15) \right.$$

$$E_{\theta} = + \frac{c\mu b}{r} I_{\omega} [krh_{1}^{(1)}(kr)]' \left\{ sin\theta , \qquad (16) \right.$$

$$H_{\phi} = + ikbI_{\omega}h_{1}^{(1)}(kr) \left\{ sin\theta . \right.$$
 (17)

Note that  $c\mu_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = Z_0$ , the impedance of free space.

Please note that this formulation is inadequate to treat the fields in the first cell anyway since we are measuring the electric field in the same mesh zone where we are driving the currents. (We will return to this point later.) We wish to do the integral

$$E(t) = \int_{-\infty}^{\infty} d\omega E(\omega) e^{-i\omega t} = c \int_{-\infty}^{\infty} dk E(k) e^{-ickt} , \qquad (18)$$

so it is necessary for us to examine the singularity structure of the integrand.

The term in brackets will have poles at the roots of

$$[kah_{1}^{(1)}(ka)]' = 0$$
, (19)

which are

$$k = \pm \frac{\sqrt{3}}{2a} - \frac{i}{2a} . {20}$$

We also have singularities at the poles of  $\boldsymbol{I}_k$ 

$$k = -i\alpha/c$$

$$k = -i\beta/c.$$
(21)

To examine the behavior at the origin we expand for small k

$$\begin{vmatrix}
j_{1}(kr) + \frac{kr}{3} \\
h_{1}^{(1)}(kr) + in_{1}(kr) + -\frac{i}{k^{2}r^{2}} \\
[krj_{1}(kr)]' + \frac{2kr}{3} \\
[krh_{1}^{(1)}(kr)]' + +\frac{i}{k^{2}r^{2}}
\end{vmatrix}$$
(22)

and find that the term in braces in (14) behaves as

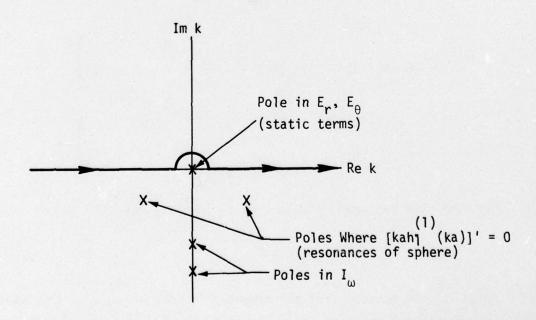
$$\left\{ \begin{array}{c} \\ \\ \end{array} \right\} \rightarrow kb \left\{ \frac{1}{3} + \frac{2}{3} \frac{a^3}{b^3} \right\} . \tag{23}$$

Since  $I_{\omega}$  is nonzero (but not singular) at the origin, we find that there are first order poles at the origin for  $E_{\mathbf{r}}$  and  $E_{\theta}$  but not for  $H_{\Phi}$ .

This is physically reasonable as it corresponds to the static fields which exist at late times in the problem.

We now turn to the behavior for large  $\,k\,$  and the evaluation of the time response by contour integration. For large  $\,k\,$ 

We choose the contour of integration from casuality considerations so that we have the static fields at late times and no fields for t < 0.



We must break time into three steps to distort our contours.

For ct < r - b we close the contour with a semicircle in the upper half-plane. The integrand on this contour behaves as

$$\frac{1}{k^4} \, e^{ik(r-b-ct)} \ \text{for} \ E_{\mathbf{r}} \ , \ E_{\theta}$$
 
$$\frac{1}{k^3} \, e^{ik(r-b-ct)} \ \text{for} \ H_{\varphi} \ ,$$

and so →0 as we let the radius become infinite. Therefore

$$E_r$$
,  $E_{\theta}$ ,  $H_{\phi} = 0$  ct < r - b.

For  $r-b \le ct \le r+b-2a$  we must break the integrand into two pieces. We see this most easily by writing the braces out in terms of  $h_1^{(1)}(kr)$  and  $h_1^{(2)}(kr)$ 

$$\left\{ \begin{array}{l} = \frac{1}{2} h_1^{(2)}(kb) - \frac{1}{2} \frac{\left[kah_1^{(2)}(ka)\right]!}{\left[kah_1^{(1)}(ka)\right]!} h_1^{(1)}(kb) \end{array} \right.$$
 (25)

These two portions of the integrand have exponential behavior like

$$e^{ik(r-ct-b)}$$
 and  $e^{ik(r-ct+b-2a)}$ ,

so that, for r-b < ct < r+b-2a, we close the contour on the first portion with a semicircle in the lower half plane, but close the second portion of the integrand in the upper half plane. This is physically reasonable, for at this time the observer at r sees the current but there is not yet sufficient time for reflections of the radiated field from the

current off of the sphere to reach the observer. Note that the portions of the integrand contributing are the same as they would be for the current without the conducting sphere. Note also that this division of terms in the integrand produce a pole at k=0 in the expression for B. This will not exist in the late time expression, ct > r - 2a + b. We encircle the poles at the origin and the poles of  $I_{(j)}$ .

For the E's we have a fourth order pole at the origin from the first term and for  $H_{\varphi}$  a third order pole at the origin.

$$\begin{split} E_{\mathbf{r}} &= + \frac{\mathrm{i}c^{2}\mu}{r^{3}b} \frac{1}{3!} \frac{\mathrm{d}^{3}}{\mathrm{d}k^{3}} \left\{ \frac{1}{\mathrm{i}ck - \beta} - \frac{1}{\mathrm{i}ck - \alpha} \right\} e^{-\mathrm{i}ckt} \bigg|_{k=0} \cos\theta \\ &+ \frac{\mathrm{c}\mu b}{r} \, h_{1}^{(1)} \, (kr) h_{1}^{(2)} \, (kb) e^{-\mathrm{i}ckt} \bigg|_{k=-\frac{\mathrm{i}\beta}{c}} \cos\theta \\ &- \frac{\mathrm{c}b}{r} \, h_{1}^{(1)} \, (kr) h_{1}^{(2)} \, (kb) e^{-\mathrm{i}ckt} \bigg|_{k=-\frac{\mathrm{i}\alpha}{c}} \cos\theta \\ &- \frac{\mathrm{c}b}{r} \, h_{1}^{(1)} \, (kr) h_{1}^{(2)} \, (kb) e^{-\mathrm{i}ckt} \bigg|_{k=0} \cos\theta \\ &- \frac{\mathrm{c}\mu b}{2r} \, \sin\theta \, \left[ \frac{1}{3!} \, \frac{\mathrm{d}^{3}}{\mathrm{d}k^{3}} \left\{ \frac{1}{\mathrm{i}ck - \beta} - \frac{1}{\mathrm{i}ck - \alpha} \right\} e^{-\mathrm{i}ckt} \bigg|_{k=0} \\ &- \frac{\mathrm{c}\mu b}{2r} \, \sin\theta \, \left[ kr h_{1}^{(1)} \, (kr) \right] \, h_{1}^{(2)} \, (kb) e^{-\mathrm{i}ckt} \bigg|_{k=-\frac{\mathrm{i}\alpha}{c}} \\ &+ \frac{\mathrm{c}\mu b}{2r} \, \sin\theta \, \left[ kr h_{1}^{(1)} \, (kr) \right] \, h_{1}^{(2)} \, (kb) e^{-\mathrm{i}ckt} \bigg|_{k=0} \\ &- \frac{\mathrm{i}kb}{2} \, \sin\theta \, h_{1}^{(1)} \, (kr) h_{1}^{(2)} \, (kb) e^{-\mathrm{i}ckt} \bigg|_{k=0} \\ &+ \frac{\mathrm{i}kb}{2} \, \sin\theta \, h_{1}^{(1)} \, (kr) h_{1}^{(2)} \, (kb) e^{-\mathrm{i}ckt} \bigg|_{k=-\frac{\mathrm{i}\beta}{c}} \\ &+ \frac{\mathrm{i}kb}{2} \, \sin\theta \, h_{1}^{(1)} \, (kr) h_{1}^{(2)} \, (kb) e^{-\mathrm{i}ckt} \bigg|_{k=-\frac{\mathrm{i}\beta}{c}} \end{aligned} \tag{28}$$

Finally we have for ct > r + b - 2a, where we only have a first order pole at the origin for  $E_r$ ,  $E_\theta$  and none at all for  $H_\phi$ , the following expressions.

$$\begin{split} \left[ \left[ \operatorname{kah}_{1}^{(1)} \left( \operatorname{ka} \right) \right]^{\prime} &= -\frac{\mathrm{i}}{\left( \operatorname{ka} \right)^{2}} \, e^{\mathrm{i} \operatorname{ka}} \left( \operatorname{ka} + \frac{\sqrt{3}}{2} + \frac{\mathrm{i}}{2} \right) \left( \operatorname{ka} - \frac{\sqrt{3}}{2} + \frac{\mathrm{i}}{2} \right) \end{split}$$
 (29) 
$$E_{\mathbf{r}} &= \frac{2c^{2} \mu b^{2}}{r^{3}} - \left[ \frac{1}{3} + \frac{2}{3} \, \frac{a^{3}}{b^{3}} \right] \, \cos \theta \left\{ \frac{1}{1 \, \operatorname{ck} - \beta} - \frac{1}{1 \, \operatorname{ck} - \alpha} \right\} \frac{k^{2} h_{1}^{(1)} \left( \operatorname{kr} \right) h_{1}^{(1)} \left( \operatorname{kb} \right) \left[ \operatorname{kah}_{1}^{(2)} \left( \operatorname{ka} \right) \right]^{\prime}}{\mathrm{ka} - \frac{\sqrt{3}}{2} + \mathrm{i} / 2} \\ &\times e^{-\mathrm{i} \operatorname{ck} t - \mathrm{i} \operatorname{ka}} \right|_{\mathbf{k}} = -\frac{\sqrt{3}}{2a} - \frac{\mathrm{i}}{2a} \\ &+ \frac{c^{2} a \mu b}{r} \, \cos \theta \left\{ \frac{1}{\mathrm{i} \, \operatorname{ck} - \beta} - \frac{1}{\mathrm{i} \, \operatorname{ck} - \alpha} \right\} \frac{k^{2} h_{1}^{(1)} \left( \operatorname{kr} \right) h_{1}^{(1)} \left( \operatorname{kb} \right) \left[ \operatorname{kah}_{1}^{(2)} \left( \operatorname{ka} \right) \right]^{\prime}}{\mathrm{ka} + \frac{\sqrt{3}}{2} + \mathrm{i} / 2} \\ &\times e^{-\mathrm{i} \operatorname{ck} t - \mathrm{i} \operatorname{ka}} \right|_{\mathbf{k}} = + \frac{\sqrt{3}}{2a} - \frac{\mathrm{i}}{2a} \\ &+ \frac{c \mu b}{r} \, \cos \theta \, h_{1}^{(1)} \left( \operatorname{kr} \right) \left\{ h_{1}^{(2)} \left( \operatorname{kb} \right) - \frac{\left[ \operatorname{kah}_{1}^{(2)} \left( \operatorname{ka} \right) \right]^{\prime}}{\left[ \operatorname{kah}_{1}^{(1)} \left( \operatorname{ka} \right) \right]^{\prime}} h_{1}^{(1)} \left( \operatorname{kb} \right) \right\} \\ &\times e^{-\mathrm{i} \operatorname{ck} t} \bigg|_{\mathbf{k}} = - \frac{\mathrm{i} \beta}{c} \\ &- \frac{c \mu b}{r} \, \cos \theta \, h_{1}^{(1)} \left( \operatorname{kr} \right) \left\{ h_{1}^{(2)} \left( \operatorname{kb} \right) - \frac{\left[ \operatorname{kah}_{1}^{(2)} \left( \operatorname{ka} \right) \right]^{\prime}}{\left[ \operatorname{kah}_{1}^{(1)} \left( \operatorname{ka} \right) \right]^{\prime}} h_{1}^{(1)} \left( \operatorname{kb} \right) \right\} \\ &\times e^{-\mathrm{i} \operatorname{ck} t} \bigg|_{\mathbf{k}} = - \frac{\mathrm{i} \alpha}{c} \end{aligned} , \tag{30}$$

$$\begin{split} E_{\theta} &= \frac{c^{2}\mu b^{2}}{r^{3}} \frac{1}{(\alpha} - \frac{1}{\beta}] \left(\frac{1}{5} + \frac{2}{3} \frac{a^{3}}{b^{3}}\right) \sin\theta \\ &- \frac{c^{2}a\mu b}{2r} \sin\theta \left\{\frac{1}{ick - \beta} - \frac{1}{ick - \alpha}\right\} \frac{k^{2}[krh_{1}^{(1)}(kr)]!h_{1}^{(1)}(kb)[kah_{1}^{(2)}(ka)]!}{ka - \frac{\sqrt{3}}{2} + i/2} \\ &\times e^{-ickt - ika} \Big|_{k = -\frac{\sqrt{3}}{2a} - \frac{i}{2a}} \\ &- \frac{c^{2}a\mu b}{2r} \sin\theta \left\{\frac{1}{ick - \beta} - \frac{1}{ick - \alpha}\right\} \frac{k^{2}[krh_{1}^{(1)}(kr)]!h_{1}^{(1)}(kb)[kah_{1}^{(2)}(ka)]!}{ka + \frac{\sqrt{3}}{2} + i/2} \\ &\times e^{-ickt - ika} \Big|_{k = -\frac{\sqrt{3}}{2a} - \frac{i}{2a}} \\ &- \frac{c\mu b}{2r} \sin\theta [krh_{1}^{(1)}(kr)]! \left\{h_{1}^{(2)}(kb) - \frac{[kah_{1}^{(2)}(ka)]!}{[kah_{1}^{(1)}(ka)]!} h_{1}^{(1)}(kb)\right\} \\ &\times e^{-ickt} \Big|_{k = -\frac{i\beta}{c}} \\ &+ \frac{c\mu b}{2r} \sin\theta [krh_{1}^{(1)}(kr)]! \left\{h_{1}^{(2)}(kb) - \frac{[kah_{1}^{(2)}(ka)]!}{[kah_{1}^{(1)}(ka)]!} h_{1}^{(1)}(kb)\right\} \\ &\times e^{-ickt} \Big|_{k = -\frac{i\alpha}{c}} , \end{split}$$

$$\begin{split} H_{\varphi} &= -\frac{\mathrm{i} b c a}{2} \sin \theta \left\{ \frac{1}{\mathrm{i} c k - \beta} - \frac{1}{\mathrm{i} c k - \alpha} \right\} \frac{k^{3} h_{1}^{(1)} \left( \mathrm{kr} \right) h_{1}^{(1)} \left( \mathrm{kb} \right) \left[ \mathrm{kah}_{1}^{(2)} \left( \mathrm{ka} \right) \right]^{\prime}}{\mathrm{ka} - \frac{\sqrt{3}}{2} + \mathrm{i}/2} \\ &\times e^{-\mathrm{i} c k t - \mathrm{i} k a} \bigg|_{k = -\frac{\sqrt{3}}{2a} - \frac{\mathrm{i}}{2a}} \\ &- \frac{\mathrm{i} b c a}{2} \sin \theta \left\{ \frac{1}{\mathrm{i} c k - \beta} - \frac{1}{\mathrm{i} c k - \alpha} \right\} \frac{k^{3} h_{1}^{(1)} \left( \mathrm{kr} \right) h_{1}^{(1)} \left( \mathrm{kb} \right) \left[ \mathrm{kah}_{1}^{(2)} \left( \mathrm{ka} \right) \right]^{\prime}}{\mathrm{ka} + \frac{\sqrt{3}}{2} + \mathrm{i}/2} \\ &\times e^{-\mathrm{i} c k t - \mathrm{i} k a} \bigg|_{k = +\frac{\sqrt{3}}{2a} - \frac{\mathrm{i}}{2a}} \\ &\cdot \frac{\mathrm{i} b}{2} \sin \theta \mathrm{kh}_{1}^{(1)} \left( \mathrm{kr} \right) \left\{ h_{1}^{(2)} \left( \mathrm{kb} \right) - \frac{\left[ \mathrm{kah}_{1}^{(2)} \left( \mathrm{ka} \right) \right]^{\prime}}{\left[ \mathrm{kah}_{1}^{(1)} \left( \mathrm{ka} \right) \right]^{\prime}} h_{1}^{(1)} \left( \mathrm{kb} \right) \right\} \\ &\times e^{-\mathrm{i} c k t} \bigg|_{k = -\frac{\mathrm{i} \beta}{c}} \\ &+ \frac{\mathrm{i} b}{2} \sin \theta \mathrm{kh}_{1}^{(1)} \left( \mathrm{kr} \right) \left\{ h_{1}^{(2)} \left( \mathrm{kb} \right) - \frac{\left[ \mathrm{kah}_{1}^{(2)} \left( \mathrm{ka} \right) \right]^{\prime}}{\left[ \mathrm{kah}_{1}^{(1)} \left( \mathrm{ka} \right) \right]^{\prime}} h_{1}^{(1)} \left( \mathrm{kb} \right) \right\} \\ &\times e^{-\mathrm{i} c k t} \bigg|_{k = -\frac{\mathrm{i} \alpha}{c}} \end{aligned} \tag{32}$$

As a check of algebra, we will calculate the late-time static behavior of E and E for b = a. (H will decay to zero at late times.) The late time fields are given in terms of the dipole moment as

$$E_{\mathbf{r}} = \frac{2D_{z}}{4\pi\epsilon_{0}\mathbf{r}^{3}}\cos\theta$$

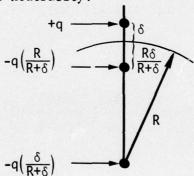
$$E_{\theta} = \frac{D_{z}}{4\pi\epsilon_{0}\mathbf{r}^{3}}\sin\theta , \qquad (33)$$

where

$$D_{z} = \int dV \rho z = \int dt \int dV \vec{J} \cdot \hat{z}$$

$$= \int dt \int dV J_{r} \cos\theta , \qquad (34)$$

as we have no  $J_{\theta}$ . We now use a quasistatic calculation to relate the dipole moment induced on the sphere to that of the charge distribution in space. We move a charge +q an infinitesimal distance from the surface, leaving a -q at the point on the surface from which the +q was emitted. The dipole moment from the motion of charge in space is  $q\delta$ . However the sphere will polarize and change this dipole moment. We can treat this polarization by using image charges; a charge at the center of the sphere must be added to preserve total charge neutrality.



The dipole moment for this set of charges is

$$D_{sph} = q\delta + q\delta \left(\frac{R}{R+\delta}\right)^2 + q\delta \left(\frac{R}{R+\delta}\right)$$

$$\delta \to 0 \quad 3q\delta \quad , \tag{35}$$

so that the effect of the polarization of the sphere is to triple the total dipole moment when compared with that of just the charge motion in space. Calculating the latter dipole moment from (1) and (34)

$$D = \left(\frac{1}{\alpha} - \frac{1}{\beta}\right) \left(\frac{4\pi}{3} b^2\right), \tag{36}$$

so that the late time fields should be

$$E_{\mathbf{r}} = \frac{2b^{2}}{\varepsilon_{0}\mathbf{r}^{3}} \left(\frac{1}{\alpha} - \frac{1}{\beta}\right) \cos\theta ,$$

$$E_{\theta} = \frac{b^{2}}{\varepsilon_{0}\mathbf{r}^{3}} \left(\frac{1}{\alpha} - \frac{1}{\beta}\right) \sin\theta ,$$
(37)

which is what we obtained as  $c^2 \mu_0 = 1/\epsilon_0$ .

When we utilize these results for the comparison with the code, we must do two things:

- 1. Specify the driver in terms of the code dipole strength. By consideration of Stokes' theorem we see that the driver in the code is not a delta function in r but a finite length  $\Delta r$ . This means that our far fields should be multiplied by  $\Delta r$ . The dispersive effects in finite difference calculations prevent an accurate calculation of effects when  $\chi$  is much smaller than the mesh cell dimensions in the direction of propagation.
- 2. Specify how to calculate the radial electric field in the innermost mesh. To compare with the finite differencing scheme, we must consider the finite length of the driver and the mesh. The prescription that we choose to handle this is to examine our fields in the limit a → b and take H<sub>Φ</sub> from the outside limit. We find the first mesh cell E<sub>r</sub> from the outside field with a correction term arising from the charge density

$$E_r = E_r(r \to b) - f(t) \cos\theta , \qquad (38)$$

where

$$f(t) = \frac{1}{\varepsilon_0} \int_0^t dt' I(t')$$

$$= \frac{1}{\varepsilon_0 \alpha} \left\{ 1 - e^{-\alpha t} \right\} - \frac{1}{\varepsilon_0 \beta} \left\{ 1 - e^{-\beta t} \right\} .$$
(39)

Finally we write the equations in the units used in MODE. The excitation factor must be converted by

$$1 \text{ Abamp/cm}^2 = 10^5 \text{ Amps/m}^2$$
, (40)

and the fields by

1 statvolt/cm = 
$$3 \times 10^4$$
 volts/m  
1 gauss =  $79.58$  Amps/m , (41)

so we define

$$F_{1} = 7.41 \times 10^{3}$$

$$F_{2} = 10^{5}$$

$$G_{1} = 3.33 \times 10^{-5}$$

$$G_{2} = 1.26 \times 10^{-2}$$

$$(42)$$

For fields outside the source region

$$(E_{\mathbf{r}}^{\text{esu}}, E_{\theta}^{\text{esu}}) = F_1 G_1 (E_{\mathbf{r}}, E_{\theta})$$

$$H_{\phi}^{\text{esu}} = F_1 G_2 H_{\phi}, \qquad (43)$$

and for fields in the first cell.

$$E_{\mathbf{r}}^{\text{esu}} = F_1 G_1 E_{\mathbf{r}} - F_2 G_1 \mathbf{f}(\mathbf{t}) \cos \theta$$

$$H_{\phi}^{\text{esu}} = F_1 G_2 H_{\phi} . \tag{44}$$

## SECTION 3

## RESULTS AND COMPARISON WITH MODE

In this section we present graphs of the electric and magnetic fields previously calculated. The surface radial and magnetic fields need no explanation; the theta electric field is shown at 1.074 meters as that is the first mesh point at which this field is non-vanishing in MODE. The fields are also shown at 2 meters which is the outer boundary in MODE. In Table 1 we present a comparison between the analytic calculation and the finite difference of the values and times of various features seen in Figures 1 through 6. The peaks and dips are broad so that the times given are not too relevant.

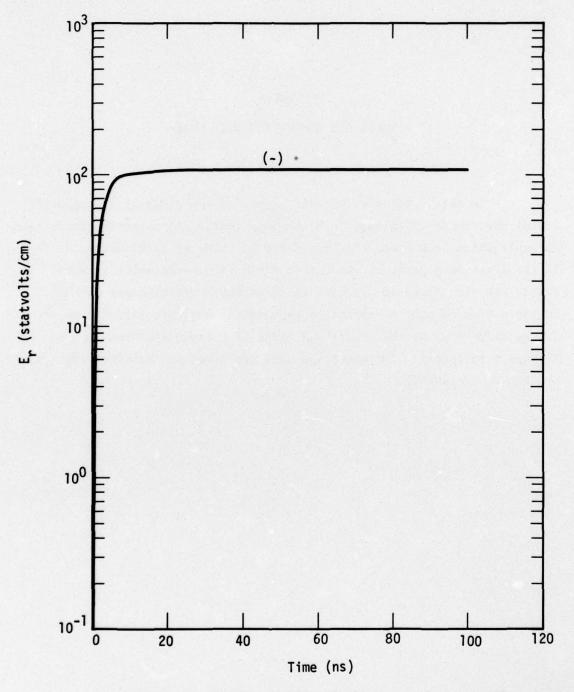


Figure 1. Radial electric field at r = 1 m,  $\theta = 0^{\circ}$ .

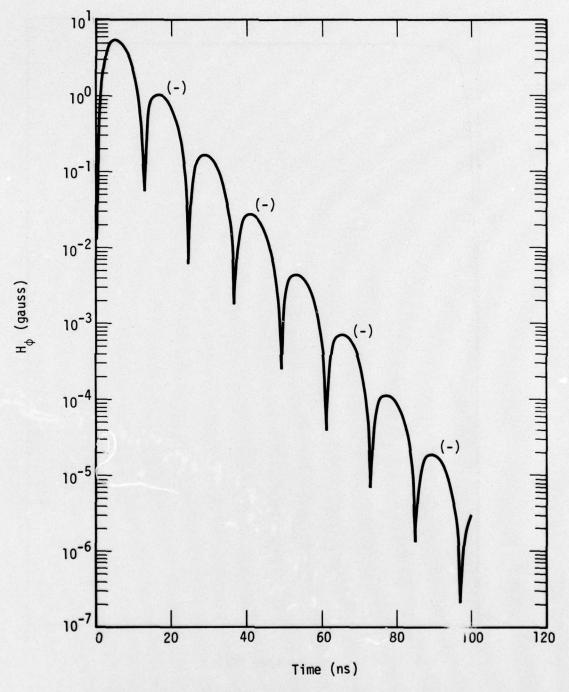


Figure 2. Azimuthal magnetic field at r = 1 m,  $\theta = 90^{\circ}$ .

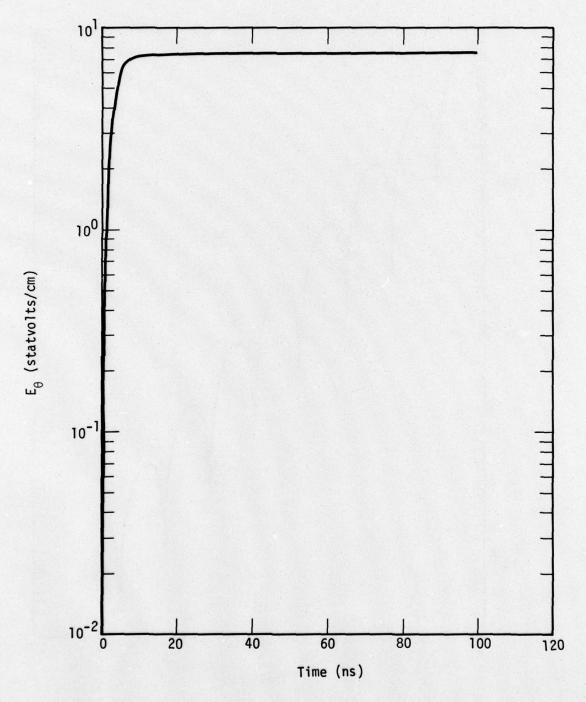


Figure 3. Theta electric field at r = 1.074 m,  $\theta = 90^{\circ}$ .

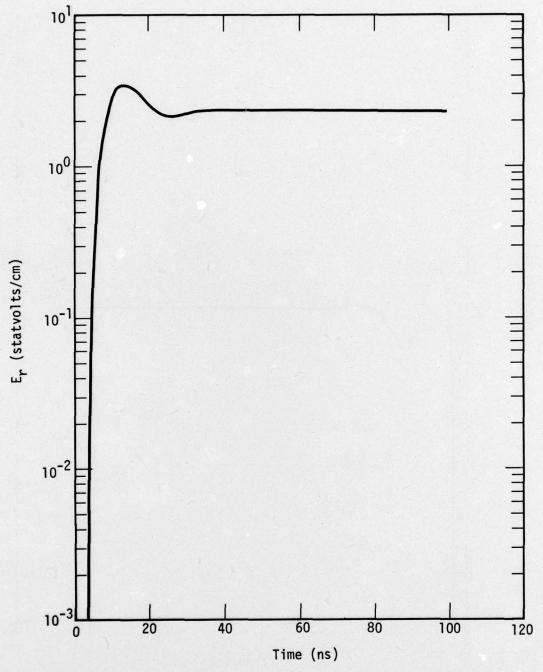


Figure 4. Radial electric field at r = 2 m,  $\theta = 0^{\circ}$ .

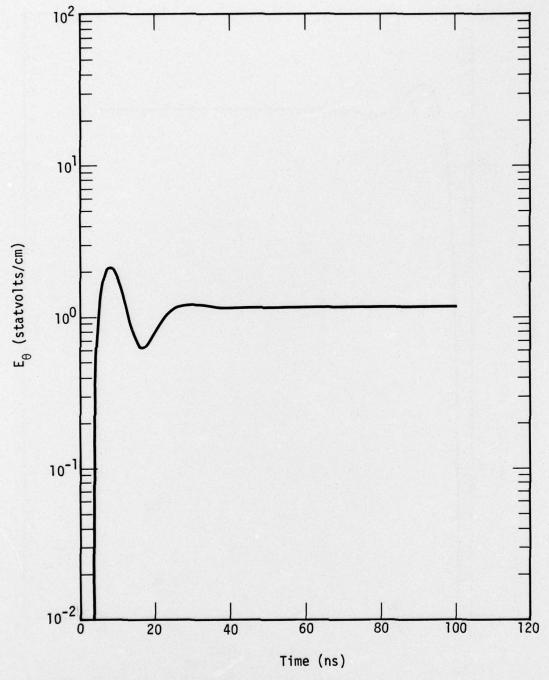


Figure 5. Theta electric field at r = 2 m,  $\theta = 90^{\circ}$ .

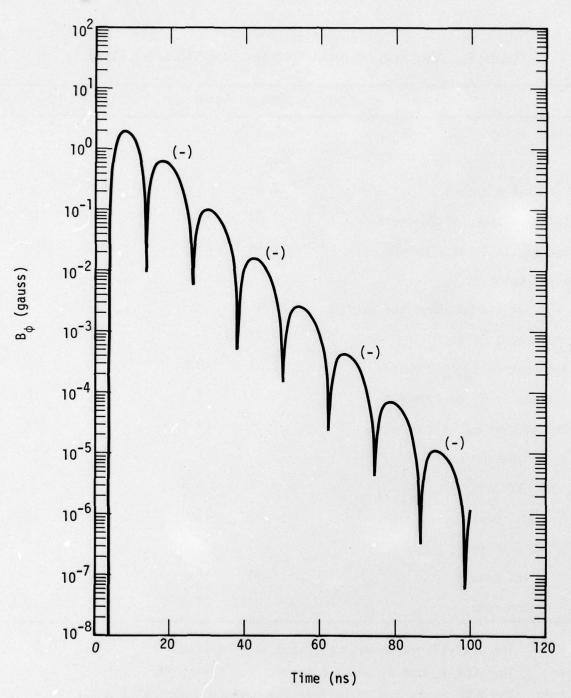


Figure 6. Azimuthal magnetic field at r = 2 m,  $\theta = 90^{\circ}$ .

Table 1. Comparison between MODE and analytic calculation.

	MODE	Time (ns)	Analytic	Time (ns)*
Final value of E <sub>r</sub>				
-at the surface**	-109.	-	-107.	-
-at 2 meters	2.33	-	2.32	-
1st peak at E <sub>r</sub> at 2 meters	3.48	13.4	3.49	13.5
1st dip in E <sub>r</sub> at 2 meters	2.18	25.5	2.14	26
Final value of $E_{\theta}$				
-at 7.4 cm above the surface	7.49	-	7.50	-
-at 2 meters***	1.23	-	1.16	_
1st peak of E <sub>0</sub> at 2 meters	2.22	8.0	2.15	8
1st dip in $E_{\theta}$ at 2 meters	0.671	16.5	0.626	17
2nd peak at $\mathbf{E}_{\theta}$ at 2 meters	1.31	28.4	1.25	29
H, at the surface**				
1st peak	5.06	5.2	5.37	5
2nd peak	-1.07	16.0	-1.04	16.5
H <sub>A</sub> at 2 meters				
1st peak	1.98	7.6	2.00	7.5
2nd peak	625	17.5	617	17.5

The analytic value was tabulated in 1 ns steps.

The MODE  $\rm E_r$  and  $\rm H_{\varphi}$  are 3.8 cm out from the surface. The MODE  $\rm E_{\theta}$  is 3.8 cm in from the outer boundary at 2 m.

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